

1. Solve the equation

$$2 \cosh^2 x - 3 \sinh x = 1$$

giving your answers in terms of natural logarithms.

(6)

$$c^2 - s^2 = 1 \Rightarrow c^2 = 1 + s^2$$

$$\therefore 2c^2 - 3s = 1$$

$$\Rightarrow 2(1 + s^2) - 3s = 1$$

$$\therefore 2 + 2s^2 - 3s = 1$$

$$\therefore 2s^2 - 3s + 1 = 0$$

$$\therefore (s-1)(2s-1) = 0$$

$$\Rightarrow \sinh x = 1 \Rightarrow x = \operatorname{arsinh} 1 = \ln(1 + \sqrt{2})$$

$$\sinh x = \frac{1}{2} \Rightarrow x = \operatorname{arsinh} \left(\frac{1}{2}\right) = \ln\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)$$



2. A curve has equation

$$y = \cosh x, \quad 1 \leq x \leq \ln 5$$

Find the exact length of this curve. Give your answer in terms of  $e$ .

(5)

$$y = \cosh x \Rightarrow \frac{dy}{dx} = \sinh x$$

$$\text{length} = \int_1^{\ln 5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$c^2 - s^2 = 1$$

$$= \int_1^{\ln 5} \sqrt{1 + \sinh^2 x} dx$$

$$= \int_1^{\ln 5} \cosh x dx$$

$$= \left[ \sinh x \right]_1^{\ln 5}$$

$$= \sinh(\ln 5) - \sinh(1)$$

$$= \frac{e^{\ln 5} - e^{-\ln 5}}{2} - \frac{e^1 - e^{-1}}{2}$$

$$= \frac{5 - \frac{1}{5}}{2} - \frac{e - \frac{1}{e}}{2}$$

$$= \frac{5 - \frac{1}{5} - e + \frac{1}{e}}{2} \times \frac{5e}{5e} = \frac{25e - e^2 - 5e^2 + 5}{10e}$$



$$= \frac{24e - 5e^2 + 5}{10e}$$

3.

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

- (a) Find the eigenvalues of A. (5)
- (b) Find a normalised eigenvector for each of the eigenvalues of A. (5)
- (c) Write down a matrix P and a diagonal matrix D such that  $P^T A P = D$ . (2)

$$(a) A - \lambda I = \begin{pmatrix} 2-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = 0 \Rightarrow (2-\lambda)((2-\lambda)^2 - 1) - (2-\lambda) = 0$$

$$\Rightarrow \Rightarrow (2-\lambda)^3 - (2-\lambda) - (2-\lambda) = 0$$

$$\therefore (2-\lambda)^3 - 2(2-\lambda) = 0$$

$$\therefore (2-\lambda)((2-\lambda)^2 - 2) = 0$$

$$\therefore (2-\lambda)(\lambda^2 - 4\lambda + 2) = 0$$

$$\therefore \cancel{(2-\lambda)} \Rightarrow 2-\lambda = 0 \therefore \lambda = 2$$

$$(2-\lambda)^2 - 2 = 0 \Rightarrow 2-\lambda = \pm\sqrt{2}$$

$$\therefore \lambda = 2 \pm \sqrt{2}$$

$$\therefore \text{E. Values are } \lambda = 2$$

$$\lambda = 2 \pm \sqrt{2}$$



## Question 3 continued

(b) Consider  $\lambda = 2$ :

$$Ax = \lambda x$$

$$\therefore \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$$\therefore \begin{pmatrix} 2x + y \\ x + 2y + z \\ y + 2z \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} \Rightarrow \begin{matrix} y = 0 \\ x + z = 0 \\ y = 0 \end{matrix}$$

$$x = -z, \quad y = 0$$

$$\text{Let } x = 1, \quad z = -1, \quad y = 0$$

$$\therefore \text{E. Vector } x = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\text{Normalised } \underline{x} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Consider  $\lambda = 2 + \sqrt{2}$ :

$$\begin{pmatrix} 2x + y \\ x + 2y + z \\ y + 2z \end{pmatrix} = \begin{pmatrix} (2 + \sqrt{2})x \\ (2 + \sqrt{2})y \\ (2 + \sqrt{2})z \end{pmatrix}$$





$$\Rightarrow \begin{cases} y = x\sqrt{2} \\ x+z = y\sqrt{2} \\ y = z\sqrt{2} \end{cases} \left. \begin{array}{l} y=1 \Rightarrow x = \frac{1}{\sqrt{2}} \Rightarrow z = \frac{1}{\sqrt{2}} \\ y=\sqrt{2} \Rightarrow x=1 \Rightarrow z=1 \end{array} \right\} \therefore x = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

$$\therefore y = x\sqrt{2} = z\sqrt{2}$$

~~2~~  ~~$\frac{y}{\sqrt{2}} = \frac{z}{\sqrt{2}}$~~   ~~$z = y$~~   ~~$z = y\sqrt{2}$~~   $\therefore$  normalise:

$$x = \frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

~~$x = \frac{y}{\sqrt{2}}$~~

$x = 2 - \sqrt{2}$

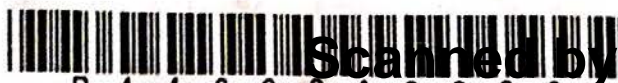
$$\Rightarrow \begin{pmatrix} 2x+y \\ x+z+y+z \\ y+z+z \end{pmatrix} = \begin{pmatrix} 2x - x\sqrt{2} \\ 2y - y\sqrt{2} \\ 2z - z\sqrt{2} \end{pmatrix}$$

$$\therefore \begin{cases} y = -x\sqrt{2} \\ x+z = -y\sqrt{2} \\ y = -z\sqrt{2} \end{cases}$$

$$\therefore y = -z\sqrt{2} = -x\sqrt{2}$$

$$y = \sqrt{2} \Rightarrow z = -1 \Rightarrow x = -1$$

$$\therefore x = \begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix} \Rightarrow \text{normalise } x = \frac{1}{2} \begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}$$





Question 3 continued

$$\therefore \lambda = 2 \Rightarrow x_n = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\lambda = 2 - \sqrt{2} \Rightarrow x_n = \frac{1}{2} \begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}$$

$$\lambda = 2 + \sqrt{2} \Rightarrow x_n = \frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

$$(c) P = \begin{pmatrix} -1/2 & 1/\sqrt{2} & 1/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ -1/2 & -1/\sqrt{2} & 1/2 \end{pmatrix}$$

~~$$P^{-1} A P = \begin{pmatrix} -1/2 & \sqrt{2}/2 & -1/2 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/2 & \sqrt{2}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$~~

~~$$= \begin{pmatrix} \frac{-2+\sqrt{2}}{2} & & \\ & 2 & \\ & & 2+\sqrt{2} \end{pmatrix}$$~~

$$D = \begin{pmatrix} 2-\sqrt{2} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2+\sqrt{2} \end{pmatrix}$$

(Total 12 marks)

Q3



4. The curve  $C$  has equation

$$y = \frac{1}{\sqrt{x^2 + 2x - 3}}, \quad x > 1$$

(a) Find  $\int y dx$  (3)

The region  $R$  is bounded by the curve  $C$ , the  $x$ -axis and the lines with equations  $x = 2$  and  $x = 3$ . The region  $R$  is rotated through  $2\pi$  radians about the  $x$ -axis.

(b) Find the volume of the solid generated. Give your answer in the form  $p\pi \ln q$ , where  $p$  and  $q$  are rational numbers to be found. (4)

(a). 
$$\int \frac{1}{\sqrt{(x+1)^2 - 4}} dx$$

$$= \operatorname{arcosh} \left( \frac{x+1}{2} \right) + C$$

(b) 
$$V = \pi \int_2^3 y^2 dx$$

$$\therefore V = \pi \int_2^3 \frac{1}{(x+1)^2 - 4} dx$$

$$= \pi \left[ \frac{1}{4} \ln \left| \frac{x-2}{x+3} \right| \right]_2^3$$

$$= \pi \left[ \frac{1}{4} \ln \left( \frac{1}{3} \right) - \frac{1}{4} \ln \left( \frac{1}{5} \right) \right]$$

$$= \frac{1}{4} \pi \ln \left( \frac{5}{3} \right)$$





5. The points  $A$ ,  $B$  and  $C$  have position vectors  $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ ,  $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$  respectively.

(a) Find a vector equation of the straight line  $AB$ .

(2)

(b) Find a cartesian form of the equation of the straight line  $AB$ .

(2)

The plane  $\Pi$  contains the points  $A$ ,  $B$  and  $C$ .

(c) Find a vector equation of  $\Pi$  in the form  $\mathbf{r} \cdot \mathbf{n} = p$ .

(4)

(d) Find the perpendicular distance from the origin to  $\Pi$ .

(2)

$$5(a) \quad \vec{AB} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$$

$$\vec{r} = \lambda \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$(b) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1-2\lambda \\ 3-3\lambda \\ 2+\lambda \end{pmatrix}$$

$$\star \frac{1-x}{2} = \lambda$$

$$\frac{3-y}{3} = \lambda$$

$$2-z = \lambda$$

$$\therefore \frac{1-x}{2} = \frac{3-y}{3} = 2-z$$

$$(x \ 6) \Rightarrow 3-3x = 6-2y = 2-6z$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \leftarrow \begin{matrix} \text{---} \\ \text{---} \end{matrix}$$



Question 5 continued

$$c) AB = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$$

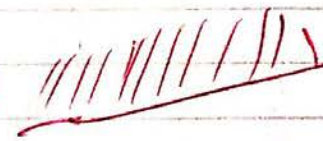
$$AC = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$$

$$\therefore AB \times AC = \begin{matrix} -2 & -3 & -1 & -2 & -3 & -1 \\ & 1 & -2 & -2 & 1 & -2 & -2 \end{matrix}$$

$$= \begin{pmatrix} 4 \\ -5 \\ 7 \end{pmatrix}$$

$$\therefore \hat{n} \cdot \begin{pmatrix} 4 \\ -5 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -5 \\ 7 \end{pmatrix} = 8 - 5 = 3$$

$$\therefore \hat{n} \cdot \begin{pmatrix} 4 \\ -5 \\ 7 \end{pmatrix} = 3 \quad \text{--- ~~--- ~~---~~ ---~~$$

(d) 

$$\hat{n} = \frac{\sqrt{10}}{30} \begin{pmatrix} 4 \\ -5 \\ 7 \end{pmatrix}$$

$$\hat{n} \cdot \hat{n} = \frac{\sqrt{10}}{30} \times 3 = \frac{\sqrt{10}}{10}$$

$$\therefore d = \frac{\sqrt{10}}{10}$$

0°



6. The hyperbola  $H$  is given by the equation  $x^2 - y^2 = 1$

(a) Write down the equations of the two asymptotes of  $H$ . (1)

(b) Show that an equation of the tangent to  $H$  at the point  $P$  ( $\cosh t, \sinh t$ ) is

$$y \sinh t = x \cosh t - 1 \quad (3)$$

The tangent at  $P$  meets the asymptotes of  $H$  at the points  $Q$  and  $R$ .

(c) Show that  $P$  is the midpoint of  $QR$ . (3)

(d) Show that the area of the triangle  $OQR$ , where  $O$  is the origin, is independent of  $t$ . (3)

6(a).  $x = \pm y \quad \therefore \begin{matrix} x = y \\ x = -y \end{matrix}$

(b)  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cosh t}{\sinh t} = \coth(t)$

$\therefore y - y_1 = m(x - x_1)$

$\therefore y - \sinh t = \frac{\cosh t}{\sinh t} x - \frac{\cosh^2 t}{\sinh t}$

$\therefore y \sinh t - \sinh^2 t = x \cosh t - \cosh^2 t$

$\therefore y \sinh t = x \cosh t + \sinh^2 t - \cosh^2 t$

$\therefore y \sinh t = x \cosh t - (\cosh^2 t - \sinh^2 t)$

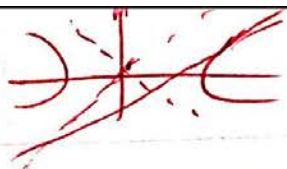
$\therefore y \sinh t = x \cosh t - 1$

as required.





Question 6 continued

~~(b)~~(b) Consider  $y = x$ 

$$\Rightarrow x \sinh t = x \cosh t - 1$$

$$\therefore (\sinh t - \cosh t)x = -1$$

$$\therefore x = \frac{1}{\cosh t - \sinh t} = y$$

$$\therefore \text{Let } \mathcal{Q} \left( \frac{1}{\cosh t - \sinh t}, \frac{1}{\cosh t - \sinh t} \right)$$

$$y = -x \Rightarrow -x \sinh t = x \cosh t - 1$$

$$\therefore x (\cosh t + \sinh t) = 1$$

$$\therefore x = \frac{1}{\cosh t + \sinh t}$$

$$\therefore \text{Let } \mathcal{R} = \left( \frac{1}{\cosh t + \sinh t}, \frac{-1}{\cosh t + \sinh t} \right)$$

## Question 6 continued

Midpoint of QR:

$$\left( \frac{1}{2} \left( \frac{1}{\cosh t + \sinh t} + \frac{1}{\cosh t - \sinh t} \right), \frac{1}{2} \left( \frac{1}{\cosh t + \sinh t} - \frac{1}{\cosh t - \sinh t} \right) \right)$$

$$\begin{aligned} \text{Let } \sinh t &= s & c^2 - s^2 &= 1 \\ \cosh t &= c \end{aligned}$$

Consider  $x$  coordinate of midpoint QR:

$$x = \frac{1}{2} \left( \frac{1}{c+s} + \frac{1}{c-s} \right) = \frac{1}{2} \left( \frac{c-s}{(c+s)(c-s)} + \frac{(c+s)}{(c-s)(c+s)} \right)$$

$$= \frac{1}{2} \left( \frac{2c}{(c+s)(c-s)} \right) = \frac{1}{2} \left( \frac{2c}{c^2 - s^2} \right)$$

$$= \frac{1}{2} \left( \frac{2c}{1} \right) = c = \cosh t$$

$$\therefore x \text{ coordinate} = \cosh t = x_p$$

Consider  $y$  coordinate of midpoint QR:

$$y = \frac{1}{2} \left( \frac{1}{c-s} - \frac{1}{c+s} \right) = \frac{1}{2} \left( \frac{c+s}{(c-s)(c+s)} - \frac{(c-s)}{(c+s)(c-s)} \right)$$

$$= \frac{1}{2} \left( \frac{c+s - c+s}{c^2 - s^2} \right) = \frac{1}{2} \left( \frac{2s}{c^2 - s^2} \right) = \frac{1}{2} \left( \frac{2s}{1} \right)$$





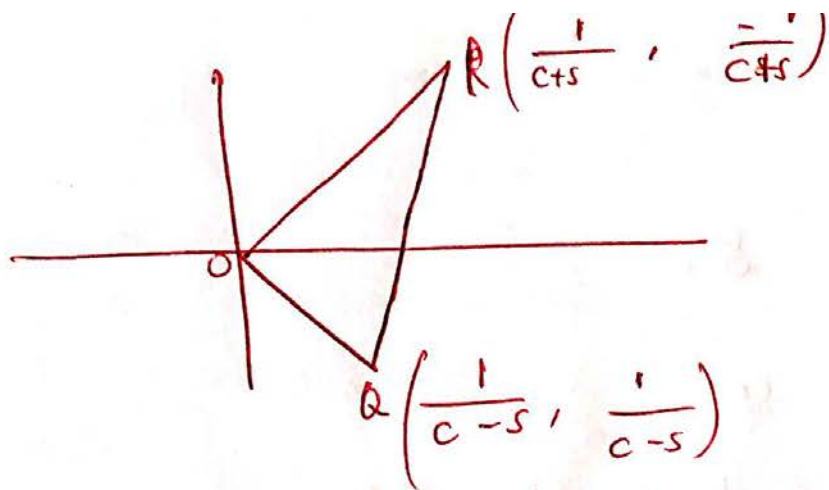
Question 6 continued

$$= s = \sin ht = y_p$$

$$\therefore \cancel{x_p = \cot}$$

$\therefore P$  is midpoint of  $QR$

(d)



Let  $c = \cosh t$   
Let  $s = \sinh t$

$$\text{Area} = \frac{1}{2} | \vec{OQ} \times \vec{OP} |$$

$$= \frac{1}{2} \cdot \sqrt{\left(\frac{1}{c-s}\right)^2 + \left(\frac{1}{c-s}\right)^2} \times \sqrt{\left(\frac{1}{c+s}\right)^2 + \left(\frac{1}{c+s}\right)^2}$$

$$= \frac{1}{2} \cdot \sqrt{2} \cdot \sqrt{\frac{1}{(c-s)^2}} \cdot \sqrt{2} \cdot \sqrt{\frac{1}{(c+s)^2}}$$

$$= \frac{1}{2} \cdot \sqrt{2} \cdot \frac{1}{c-s} \cdot \sqrt{2} \cdot \frac{1}{c+s}$$

$$= \frac{1}{(c-s)(c+s)} = \frac{1}{c^2 - s^2} = 1$$

$\therefore$  Area is always  
one regardless of how  $t$   
varies

$\therefore$  Area is independent of  $t$



7.  $I_n = \int \sin^n x \, dx, n \geq 0$

(a) Prove that for  $n \geq 2$

$$I_n = \frac{1}{n} (-\sin^{n-1} x \cos x + (n-1)I_{n-2}) \quad (4)$$

Given that  $n$  is an odd number,  $n \geq 3$

(b) show that

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{(n-1)(n-3)\dots 6.4.2}{n(n-2)(n-4)\dots 7.5.3} \quad (4)$$

(c) Hence find  $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^2 x \, dx$  (3)

$$\text{7(a)} \quad I_n = \int \sin^n x \, dx = \int \sin^{n-1} x \sin x \, dx$$

$$\text{Let } u = \sin^{n-1} x \quad u' = (n-1)(\sin^{n-2} x) \cos x$$

$$v' = \sin x \quad v = -\cos x$$

$$\therefore I_n = -\cos x \sin^{n-1} x + (n-1) \int \cos^2 x \sin^{n-2} x \, dx$$

$$\text{Since } \cos^2 x = 1 - \sin^2 x$$

$$\therefore I_n = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x - \sin^n x \, dx$$

$$\therefore I_n = -\cos x \sin^{n-1} x + (n-1) [I_{n-2} - I_n]$$

$$\therefore I_n = -\cos x \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$\therefore (1+n-1) I_n = -\cos x \sin^{n-1} x + (n-1) I_{n-2}$$

$$\therefore n I_n = -\cos x \sin^{n-1} x + (n-1) I_{n-2}$$



$$\therefore I_n = \frac{1}{n} \left( -\cos x \sin^{n-1} x + (n-1) I_{n-2} \right)$$

(b) Let  $I_n = \int_0^{\pi/2} \sin^n x \, dx$

$$I_n = \frac{1}{n} \left( \left[ -\cos x \sin^{n-1} x \right]_0^{\pi/2} + (n-1) I_{n-2} \right)$$

$$\therefore I_n = \frac{(n-1)}{n} I_{n-2}, \quad I_{n-2} = \frac{(n-3)}{n-2} I_{n-4}, \dots$$

$$\therefore I_n = \left( \frac{n-1}{n} \right) \left( \frac{n-3}{n-2} \right) I_{n-4} = \left( \frac{n-1}{n} \right) \left( \frac{n-3}{n-2} \right) \left( \frac{n-5}{n-4} \right) I_{n-6} \dots$$

$$\Rightarrow I_n = \frac{(n-1)(n-3)(n-5) \dots 6 \cdot 4 \cdot 2}{n(n-2)(n-4) \dots 7 \cdot 5 \cdot 3}$$

as required



Question 7 continued

$$(C) \int_0^{\pi/2} \sin^5 x \cos^2 x \, dx$$

$$\text{let} = \int_0^{\pi/2} \sin^5 x (1 - \sin^2 x) \, dx$$

$$= \int_0^{\pi/2} \sin^5 x - \sin^7 x \, dx$$

$$= \int_0^{\pi/2} \sin^5 x \, dx - \int_0^{\pi/2} \sin^7 x \, dx$$

$$= \frac{4 \cdot 2 \cdot 1}{5 \cdot 3 \cdot 1} - \frac{6 \cdot 4 \cdot 2}{7 \cdot 5 \cdot 3 \cdot 1}$$

$$= \frac{8}{15} - \frac{16}{35} = \frac{8}{105}$$

8. The ellipse  $E$  has equation  $x^2 + 4y^2 = 4$

(a) (i) Find the coordinates of the foci,  $F_1$  and  $F_2$ , of  $E$ .

(ii) Write down the equations of the directrices of  $E$ .

(4)

(b) Given that the point  $P$  lies on the ellipse, show that

$$|PF_1| + |PF_2| = 4$$

(4)

A chord of an ellipse is a line segment joining two points on the ellipse.

The set of midpoints of the parallel chords of  $E$  with gradient  $m$ , where  $m$  is a constant, lie on a straight line  $l$ .

(c) Find an equation of  $l$ .

(6)

$$8(a). \quad \frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4} = 1 - e^2$$

$$\therefore \frac{1}{4} = 1 - e^2 \Rightarrow e^2 = \frac{3}{4}$$

$$\therefore e = \frac{\sqrt{3}}{2}$$

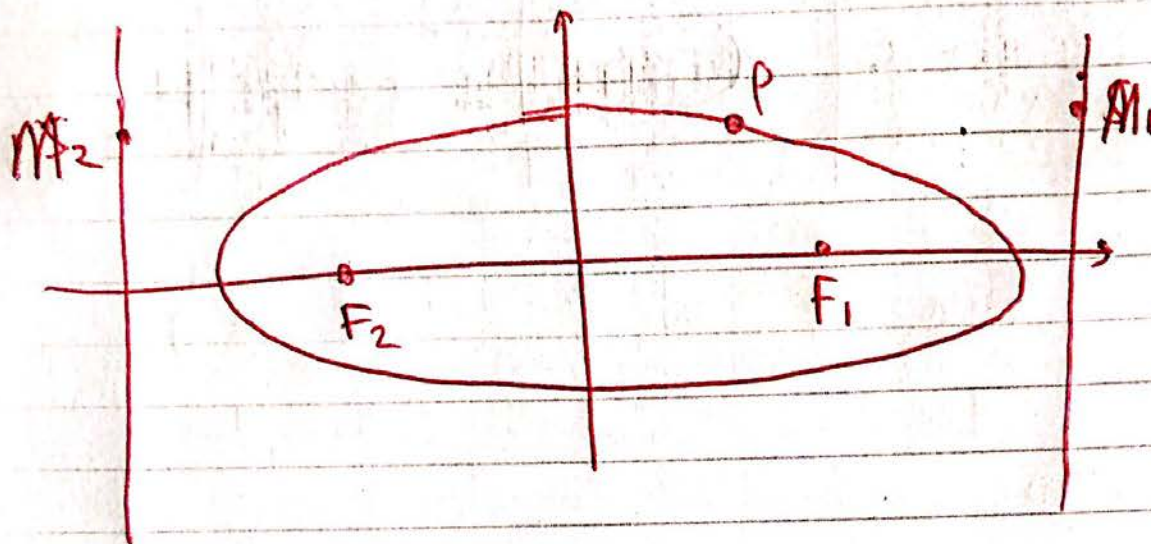
$$(i) \quad F_1 (\sqrt{3}, 0)$$

$$F_2 (-\sqrt{3}, 0)$$

$$(ii) \quad x = \pm \frac{2}{\frac{\sqrt{3}}{2}} = \pm \frac{4\sqrt{3}}{3}$$



## Question 8 continued



$$e = \frac{PF_1}{PM_1} \quad , \quad e = \frac{PF_2}{PM_2}$$

$$\therefore PF_1 = ePM_1 \quad \& \quad PF_2 = ePM_2$$

$$PM_1 + PM_2 = \frac{4\sqrt{3}}{3} + \frac{4\sqrt{3}}{3} = \frac{8\sqrt{3}}{3}$$

$$\therefore ePM_1 + ePM_2 = e \times \frac{8\sqrt{3}}{3}$$

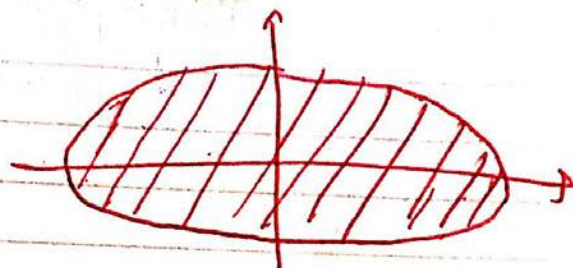
$$\therefore PF_1 + PF_2 = e \times \frac{8\sqrt{3}}{3} = \frac{\sqrt{3}}{2} \times \frac{8\sqrt{3}}{3}$$

$$\therefore PF_1 + PF_2 = 4$$

as required

$$(C) \quad x^2 + 4y^2 = 4$$

$$\therefore 2x + 8y \frac{\partial y}{\partial x} = 0$$



$$\therefore \frac{\partial y}{\partial x} = \frac{-2x}{8y} = -\frac{x}{4y}$$

$$\Rightarrow \frac{\partial y}{\partial x} = -\frac{x}{4y} = m$$

$$\therefore -x = 4ym$$

$$\therefore y = -\frac{x}{4m}$$